

For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Alvin's house is located at (3, 3, 4), Simon's is at (7, 8, 9), and Theodore's is at (11, 5, 5). What is the shortest distance Alvin can travel after leaving his own house in order to visit both of his friends' houses?

- A)  $\sqrt{69} + \sqrt{66}$     B)  $\sqrt{66} + \sqrt{41}$     C)  $\sqrt{69} + \sqrt{37}$     D)  $\sqrt{41} + \sqrt{37}$     E) NOTA

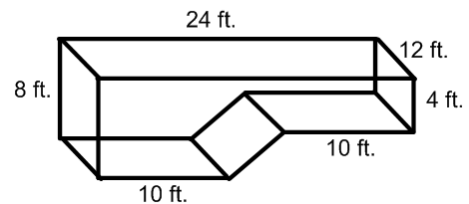
2. A particular regular prism has 24 edges. How many faces does it have?

- A) 6    B) 8    C) 10    D) 12    E) NOTA

3. A sidewalk in the shape of a rectangular prism is 4 inches thick, 3 feet wide, and 100 feet long. You have to order enough concrete to construct the sidewalk, but it only comes in increments of  $\frac{1}{2}$  cubic yard. To the nearest half of a cubic yard, how many cubic yards of concrete should be ordered to construct the entire sidewalk?

- A)  $33\frac{1}{2}$     B)  $3\frac{1}{2}$     C)  $11\frac{1}{2}$     D) 4    E) 12

4. A swimming pool is 24 ft. long, 12 ft. wide, and completely filled with water. The shallow end of the pool is 10 ft. by 12 ft., the deep end is 10 ft. by 12 ft., and there is a flat ramp descending between the two ends. If the shallow end is 4 ft. deep and the deep end is 8 ft. deep, then how many gallons of water will fit in the pool? (For the purposes of this problem, 1 cubic foot of space can hold 7.5 gallons of water.)



- A) 12,960    B) 13,680    C) 14,400    D) 15,120    E) NOTA

5. A sphere has radius 13 and is centered at the point (0, 0, 0). How many points on the sphere have all integer coordinates and at least one of the coordinates (x, y, or z) is 12?

- A) 24    B) 36    C) 48    D) 72    E) NOTA

6. A particular Platonic solid has k faces converging at each vertex. For how many values of k is this possible?

- A) 4    B) 3    C) 2    D) 1    E) NOTA

7. A sphere has radius 12. What is the ratio of its volume to its surface area?

- A) 12:1    B) 3:1    C) 6:1    D) 4:1    E) NOTA

8. A right circular cone with radius 3 and slant height 5 has a sphere inscribed in it. Find the shortest distance from the vertex of the cone to the surface of the sphere.

- A)  $\frac{2}{3}$     B)  $\frac{3}{2}$     C) 1    D)  $\frac{4}{3}$     E) NOTA

9. A rectangular prism has a front face with area  $88 \text{ un.}^2$  and perimeter 48 units. If the volume of the prism is  $1320 \text{ un.}^3$ , find the length of the space diagonal of the prism.

- A) 15                      B) 25                      C)  $12\sqrt{2}$                       D)  $\frac{8\sqrt{131}}{3}$                       E) NOTA

10. Two fair, six-sided dice are rolled landing on a flat surface. Assuming the dice are not touching, what is the probability that all of the exposed faces have a sum greater than 38?

- A)  $\frac{1}{4}$                       B)  $\frac{1}{6}$                       C)  $\frac{1}{12}$                       D)  $\frac{1}{18}$                       E) NOTA

11. A cylindrical jug with base radius 3 inches and height 9 inches is two-thirds full of water. By how many inches will the water level drop after a cone-shaped glass with base radius  $1\frac{1}{2}$  inches and height of 4 inches is filled from the contents of the jug?

- A)  $\frac{1}{3}$                       B)  $\frac{1}{2}$                       C)  $\frac{3}{4}$                       D)  $\frac{3}{2}$                       E) NOTA

**For problems 12 and 13, consider a tower constructed from 6 cubes with edges 1, 2, 3, 4, 5, and 6 inches respectively. The 5 inch cube is glued to the top of the 6 inch cube, leaving a uniform strip of the top face of the 6 inch cube exposed entirely around the 5 inch cube. Then the 4 inch cube is glued on top of the 5 inch cube in like manner, and the pattern is continued until the 1 inch cube is glued atop the 2 inch cube.**

12. What is the surface area of this tower of cubes (including the bottom face of the 6 inch cube)?

- A) 386                      B) 436                      C) 486                      D) 536                      E) NOTA

13. Let A be a randomly chosen vertex on the bottom of the 6 inch cube. Let B be the vertex on top of the 1 inch cube that happens to be nearest to point A. What portion of  $\overline{AB}$  is positioned outside of the tower of cubes?

- A)  $\frac{1}{21}$                       B)  $\frac{2}{21}$                       C)  $\frac{1}{7}$                       D)  $\frac{4}{21}$                       E) NOTA

14. An equilateral triangle of side length 4 is fixed on the coordinate plane so that its incenter is at the origin, and one of its sides is perpendicular to the x-axis. Find the volume of the solid swept out by revolving this triangle around the y-axis.

- A)  $9\pi$                       B)  $10\pi$                       C)  $11\pi$                       D)  $12\pi$                       E) NOTA

15. A circle of radius 6 is cut into 3 sectors with areas in a ratio of 1:2:3. Then each sector is formed into a cone by connecting its two bounding radii. What is the volume of the cone formed from the largest sector?

- A)  $\frac{\pi\sqrt{35}}{3}$                       B)  $\frac{16\pi\sqrt{2}}{3}$                       C)  $9\pi\sqrt{3}$                       D)  $\frac{32\pi\sqrt{3}}{3}$                       E) NOTA

16. An equilateral triangle of side length 12 is folded to form a regular tetrahedron. What is the volume of this solid?

- A)  $18\sqrt{2}$       B)  $12\sqrt{2}$       C)  $18\sqrt{6}$       D)  $12\sqrt{6}$       E) NOTA

17. The surface area of a regular hexahedron is 324 square inches. What is the lateral area of this regular hexahedron?

- A) 54      B) 108      C) 162      D) 216      E) NOTA

18. A square pyramid has a base edge of length 10 and a lateral edge of length 13. What is the volume of the pyramid?

- A) 400      B)  $\frac{100\sqrt{119}}{3}$       C) 200      D)  $\frac{100\sqrt{69}}{3}$       E) NOTA

19. A solid right circular cone has a total surface area of  $144\pi$  and a lateral area of  $80\pi$ . What is the volume of the cone?

- A)  $128\pi$       B)  $\frac{512\pi}{3}$       C)  $64\pi$       D)  $\frac{256\pi}{3}$       E) NOTA

20. A right circular cone is altered so that its height is increased by 10 percent and its radius is decreased by 10 percent. What impact has this made on the volume of the cone (to the nearest tenth of a percent)?

- A) Increase 8.9%    B) Increase 10.1%    C) Decrease 8.9%    D) Decrease 10.1%    E) NOTA

21. Two similar prisms are such that the volume of the larger is 250 while the volume of the smaller is 128. If the surface area of the larger is 150, what is the surface area of the smaller?

- A) 64      B) 96      C) 112      D) 128      E) NOTA

22. An ant crawls along the surface of a cube with side length of 3 ft. What is the shortest distance the ant can travel to get from one vertex to the diametrically opposite vertex?

- A)  $3\sqrt{3}$       B) 6      C)  $3\sqrt{5}$       D)  $3\sqrt{6}$       E) NOTA

23. A cylinder has height 8 and radius 3. A double-cone is formed by attaching the bases of two identical cones, each with radius 3 and height 4. What is the ratio of the volume of the cylinder to the volume of the double cone?

- A) 2:3      B) 2:5      C) 3:2      D) 5:2      E) NOTA

24. A solid white cube of side length 5 inches is dipped into blue paint and then cut into 125 separate 1 inch cubes. What is the probability that a randomly drawn 1 inch cube will be painted blue on exactly 2 sides?

- A)  $\frac{36}{125}$       B)  $\frac{8}{125}$       C)  $\frac{54}{125}$       D)  $\frac{27}{125}$       E) NOTA

25. A square pyramid with base edge 12 and height 8 is cut parallel to the base and the altitude is cut in half. What is the volume of the frustum created by this cut?

- A) 48                      B) 144                      C) 288                      D) 336                      E) NOTA

**For questions 26-28, consider a 3 in. by 4 in. by 5 in. rectangular prism that is constructed from cubes of side length 1 inch.**

26. Let A and B be diametrically opposite vertices of the rectangular prism. Larry the Larva can crawl from point A to point B, but only by traveling along the edges of the 1 inch cubes that make up the figure. If each moment of his journey always takes him closer to point B (until he reaches point B), then how many different pathways can Larry the Larva choose?

- A) 11,880                      B) 15,840                      C) 23,760                      D) 27,720                      E) NOTA

27. If Larry the Larva could crawl in a line segment from point A to point B, then how many of the 1 inch cubes will he enter during the trip?

- A) 8                      B) 10                      C) 12                      D) 14                      E) NOTA

28. What is the smallest number of additional cubes it would take to construct a protective layer of 1 inch cubes around the original prism so that the resulting solid is also a prism?

- A) 168                      B) 150                      C) 84                      D) 60                      E) NOTA

29. A water tank in the shape of a cylinder with base radius 4 feet and height 9 feet, is lying so that its axis is horizontal. If the water surface is 2 feet from the bottom of the tank, how many cubic feet of water are in the tank?

- A)  $48\pi - 18\sqrt{3}$     B)  $48\pi - 36\sqrt{3}$     C)  $32\pi - 18\sqrt{3}$     D)  $32\pi - 24\sqrt{3}$     E) NOTA

30. The volume and surface area of a hemisphere are numerically equivalent. What is the volume of the largest cube which can be inscribed in this hemisphere?

- A)  $27\sqrt{3}$                       B)  $32\sqrt{2}$                       C)  $27\sqrt{6}$                       D)  $32\sqrt{6}$                       E) NOTA